

ZZ brane decay in D dimensions

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ABSTRACT: In this paper we consider the ZZ brane decay in a D -dimensional background with a linear dilaton and a Liouville potential switched on. We mainly calculate the closed string emission rate during the decay process. For the case of a spacelike dilaton we find a similar Hagedorn behavior, in the closed string UV region, with the brane decay in the usual 26d and 2d bosonic string theory. This means that all of the energy of the original brane converts into outgoing closed strings. In the IR region the result is finite. We also give some comments about the case that the dilaton is null.

KEYWORDS: D-branes, Conformal Field Models in String Theory, Tachyon Condensation.

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1. Introduction

String theory in time-dependent backgrounds is one of the most important questions that need to be understood. Dealing with general time-dependent backgrounds is still out of our abilities. Therefore simple solvable models, capturing some important aspects of the realistic physics, are laboratories for theoretical ideas and tools, which may give us some valuable lessons in more general and more difficult cases.

Among these the open string tachyon condensation, initiated by Sen [1–3], is a class of important models which are intensely studied in the past few years. For a comprehensive review in this subject with a complete list of references, see [4]. These works give us many surprises and insights into the tachyon physics. When coupling to the closed strings, a time-dependent open string field configuration, such as the rolling tachyon solution, on an unstable D-brane acts as a time-dependent source of various closed string fields, and produces closed string radiation [5–9]. By studying various emission processes, people realize that there is a new kind of open/closed string duality [10]. Recently the authors of [11] carefully study the subtle points in the modular transformation which is related with unitarity and channel duality. The study of open string tachyon also leads to the “reloading of the matrix” [12, 13], which gives a deep holographic understanding of the $c = 1$ matrix model, identifying it as the world-line theory of the ZZ brane low energy dynamics. This holographic viewpoint has also been extended to the supersymmetric cases [14–16].

In the calculation of the closed string radiation rate, people find a Hagedorn divergence both in 26d [8] and 2d [13] string theory. In [9] the authors consider the background with a linear dilaton, and they find that with this deformation the ultraviolet divergence in the

closed string emission rate is absent. The linear dilaton CFT is simple enough, nontrivial yet, to be treated exactly. There are many interesting related works. In [17] the author discusses the D-brane decay in the linear dilaton background from the viewpoint of the effective dynamics, see also [18] with electromagnetic field turned on. In [19] the author makes a deformation of 2d string theory with the X^0 direction replaced by a timelike linear dilaton, and discuss its matrix model description. The linear dilaton is also related to many other important CFTs, e.g. the near throat region of the NS5 brane, the 2d black hole $SL(2, \mathbb{R})/U(1)$ etc. The brane decay in these backgrounds has been studied by [20–23] and other authors. Recently [24] studies a background which is Minkowskian in the string frame with a null linear dilaton, and proposes a dual matrix string model to describe the dynamics near the big-bang singularity. A pure linear dilaton has a bare strong coupling region. A convenient way to regularize this in the string perturbation theory is to embed it into the Liouville field theory, where the strong coupling region is effectively screened by the exponential tachyon potential. In the past few years we have gained much knowledge about the Liouville theory, especially about the D-branes in this theory. There are two types of D-branes. One is extended in the Liouville direction [25, 26], the other is localized [27] in the strong coupling region.

In this paper we study the decay process of a ZZ-type p -brane in the background

$$\mathbb{R}_t \times \mathbb{R}_L \times \mathbb{R}^{D-2}, \tag{1.1}$$

where the gradient of the dilaton has components along \mathbb{R}_t and \mathbb{R}_L . The difference of our background with that of [9] is that we turn on an exponential bulk tachyon potential along \mathbb{R}_L , making it become a Liouville direction. In presence of the linear dilaton it is illegal to impose the usual Dirichlet boundary condition, since it breaks the world-sheet conformal invariance. However having the exponential potential we can utilize the knowledge from Liouville theory to study the ZZ-type brane, which is localized in the strong coupling region of \mathbb{R}_L direction and has usual Dirichlet or Neumann boundary conditions in other spacial directions. The time direction CFT describing the brane decay is the so-called Timelike Boundary Liouville (TBL) theory, which is initially studied in [28, 29]. By tuning the gradient of the dilaton the dimension D of the spacetime varies from 2 to 26. The critical case $D = 26$ corresponds to the null dilaton. In this situation not only the open string configuration on the D-brane is time-dependent, the background in the bulk is also variant along with time.

This paper is organized as follows. In section 2 we introduce the background and review the construction of the boundary states in this background. In section 3 we calculate the closed string field produced by the brane decay. The result is, as expected, that the contribution of the linear dilaton changes the on-shell condition of the closed string field. In section 4 we calculate the imaginary part of the annulus amplitude, which is identified, using the optical theorem, as the emission rate of closed strings. We find that when $2 < D < 26$ (spacelike dilaton) there is a Hagedorn behavior in the closed string UV, which is as same as the brane decay in the usual 26d and 2d string theory [8, 13], while different from [9], although there is also a linear dilaton. Our result, as in [8], means that all of the brane energy converts into closed strings. In the closed string IR, however, the

emission rate is finite. These two aspects are the main results of this paper. We also give some comments about the case $D = 26$ which corresponds that the dilaton is null. In section 5 we make some concluding remarks. In appendix we give some calculation details.

2. Background and boundary states

In this paper we will study the decay process of a ZZ-type D-brane in the framework of the boundary CFT. The total world-sheet action is $S = S_{\text{bulk}} + S_{\text{bdy}}$. The bulk is a tensor product of a time-like linear dilaton X^0 , a Liouville direction X^1 and the residual free bosons X^i with $i = 2, \dots, D$. The boundary action is the so-called “half s-brane” deformation of the X^0 bulk CFT. To be explicit we list the total action

$$\begin{aligned}
 S &= (S_{X^0} + S_{X^1} + S_{X^i}) + S_{\text{bdy}}, \\
 S_{X^0} &= -\frac{1}{4\pi} \int d^2\sigma \sqrt{g} (g^{ab} \partial_a X^0 \partial_b X^0 + R V_0 X^0), \\
 S_{X^1} &= \frac{1}{4\pi} \int d^2\sigma \sqrt{g} (g^{ab} \partial_a X^1 \partial_b X^1 + R V_1 X^1 + 4\pi\mu e^{2bX^1}), \\
 S_{X^i} &= \frac{1}{4\pi} \int d^2\sigma \sqrt{g} g^{ab} \partial_a X^i \partial_b X^i, \\
 S_{\text{bdy}} &= \frac{1}{2\pi} \int ds g^{1/4} (K V_0 X^0 + 2\pi\lambda e^{\beta X^0}).
 \end{aligned} \tag{2.1}$$

Here g_{ab} is the world-sheet metric, R is the 2d curvature scalar, and K is the extrinsic curvature of the world-sheet boundary. From the action we see that the dilaton $\Phi = V_0 X^0 + V_1 X^1$. The X^0 part is the Timelike Boundary Liouville (TBL) with vanishing 2d cosmological constant, introduced in [28]. For the theory to be perturbatively well-defined we also turn on the bulk tachyon $T \sim e^{2bX^1}$ to make the X^1 direction to be a standard Liouville CFT. Conformal invariance requires that

$$V_0 = \beta - \frac{1}{\beta}, \quad V_1 = b + \frac{1}{b}. \tag{2.2}$$

The central charge is

$$c^{X^0} = 1 - 6V_0^2, \quad c^{X^1} = 1 + 6V_1^2. \tag{2.3}$$

The dimension D of spacetime is

$$D = 26 - 6(-V_0^2 + V_1^2). \tag{2.4}$$

The primary fields of the X^0 and X^1 CFTs are $e^{ik_0 X^0 + V_0 X^0}$ and $e^{ik_1 X^1 + V_1 X^1}$, with the conformal weights $-\frac{1}{4}(k_0^2 + V_0^2)$ and $\frac{1}{4}(k_1^2 + V_1^2)$ respectively.

To study the brane decay a central object is the boundary state of the unstable brane. Since the total CFT is a tensor product of several parts, we can construct the boundary state for each part separately, and then multiply them together. We first deal with the X^0

direction. The X^0 CFT is the so-called Timelike Boundary Liouville (TBL) theory. The one point function and the corresponding boundary state have been worked out in [28]. The TBL can be related to the spacelike boundary Liouville (SBL), defined by

$$\frac{1}{4\pi} \int d^2\sigma \sqrt{g} (g^{ab} \partial_a \phi \partial_b \phi + RQ\phi + 4\pi\mu e^{2b\phi}) + \frac{1}{2\pi} \int ds g^{1/4} (KQ\phi + 2\pi\lambda e^{b\phi}) \quad (2.5)$$

with $Q = b + \frac{1}{b}$, by the analytical continuation¹

$$X^0 \rightarrow i\phi, \quad \beta \rightarrow -ib, \quad V_0 \rightarrow -iQ. \quad (2.6)$$

Then the FZZT one point function² obtained in [25, 26] gives us its timelike counterpart

$$\begin{aligned} U_{X^0}(k_0) &= \langle e^{ik_0 X^0(0) + V_0 X^0(0)} \rangle_{\text{TBL}} = i \langle e^{-k_0 \phi(0) + Q\phi(0)} \rangle_{\text{SBL}} \\ &= -\frac{i}{2^{1/4} 2\pi b} \left[\frac{2\pi\lambda}{\Gamma(1-b^2)} \right]^{\frac{k_0}{b}} \Gamma(-k_0/b) \Gamma(1-bk_0) \\ &= -\frac{1}{2^{1/4} 2\pi\beta} \left[\frac{2\pi\lambda}{\Gamma(1+\beta^2)} \right]^{-\frac{ik_0}{\beta}} \Gamma(ik_0/\beta) \Gamma(1-i\beta k_0). \end{aligned} \quad (2.7)$$

In the second line we have taken the limit (see [28])

$$\mu \rightarrow 0, \quad s \rightarrow \infty, \quad \mu \cosh^2 \pi b s = \lambda^2 \sin \pi b^2 \quad \text{fixed} \quad (2.8)$$

to turn off the bulk cosmological constant term in the X^0 direction. Having the one point function we can construct the corresponding boundary state as

$$|B\rangle_{X^0} = \int \frac{dk_0}{2\pi} U_{X^0}(k_0) |k_0\rangle, \quad (2.9)$$

where $|k_0\rangle$ is the Ishibashi state corresponding to the primary state $|k_0\rangle = e^{ik_0 X^0 + V_0 X^0} |0\rangle$.

Now we turn to the X^1 part, which is the standard Liouville theory. In this theory there is an important boundary state [27], called ZZ brane, corresponding to the degenerate representation of the Virasoro algebra. The ZZ brane boundary state is

$$|B\rangle_{X^1} = \int \frac{dk_1}{2\pi} U_{X^1}(k_1) |k_1\rangle, \quad (2.10)$$

The unnormalized one point function is

$$\begin{aligned} U_{X^1}(k_1) &= \langle e^{ik_1 X^1(0) + V_1 X^1(0)} \rangle_{\text{ZZ}} = -\frac{2^{3/4} \pi b \tilde{\mu}^{-ik_1/2b}}{\Gamma(-ik_1/b) \Gamma(1-ibk_1)}, \\ \tilde{\mu} &= \pi\mu\gamma(b^2), \quad \gamma(b^2) = \Gamma(b^2)/\Gamma(1-b^2), \end{aligned} \quad (2.11)$$

¹Although the continuation from TBL to SBL has some subtleties [29–31], especially in the multi-point function, the naive manipulation in the one point function gives the correct result.

²Here we use the unnormalized one point function, which is just the inner product of the boundary state with the Ishibashi state.

where $|k_1\rangle\rangle$ is the Ishibashi state corresponding to the primary state $|k_1\rangle = e^{ik_1 X^1 + V_1 X^1} |0\rangle$.

The remaining parts of the boundary state corresponding to the X^i and the Fadeev-Popov ghost are standard

$$|B\rangle_{X^i} = \int \frac{d^{D-p-2} k_\perp}{(2\pi)^{D-p-2}} \exp \left[- \sum_{n=1}^{\infty} \frac{1}{n} S_{ij} \alpha_{-n}^i \tilde{\alpha}_{-n}^j \right] |k_\perp, k_\parallel = 0\rangle, \quad (2.12)$$

$$|B\rangle_{\text{gh}} = \frac{1}{2} \exp \left[\sum_{n=1}^{\infty} (c_{-n} \tilde{b}_{-n} - b_{-n} \tilde{c}_{-n}) \right] (c_0 + \tilde{c}_0) | \downarrow \rangle. \quad (2.13)$$

The momentum integration in the X^i part boundary state is only over the transverse direction k_\perp . The ghost vacuum $| \downarrow \rangle$ is defined by $c_m | \downarrow \rangle = b_n | \downarrow \rangle = 0$ for $m > 0, n \geq 0$.

The total boundary state $|B_p\rangle$ for the open string tachyon condensation on a ZZ-type Dp-brane is the tensor product of the X^0, X^1, X^i and the ghost part

$$|B_p\rangle = |B\rangle_{X^0} \otimes |B\rangle_{X^1} \otimes |B\rangle_{X^i} \otimes |B\rangle_{\text{gh}}. \quad (2.14)$$

3. Closed string field configuration

After reviewing the construction of the boundary state in the previous subsection, we now calculate the closed string field configuration produced by the ZZ-type Dp-brane, using the method of [32]. The closed string field $|\Psi\rangle$ is a state with ghost number 2 in the Hilbert space of the first quantized closed string theory, which satisfies the constraints

$$(b_0 - \tilde{b}_0)|\Psi\rangle = 0, \quad (L_0 - \tilde{L}_0)|\Psi\rangle = 0. \quad (3.1)$$

The linearized equation of motion of $|\Psi\rangle$ is

$$2(Q_B + \tilde{Q}_B)|\Psi\rangle = |B_p\rangle. \quad (3.2)$$

Here $Q_B + \tilde{Q}_B$ is the BRST charge. The string field is easier to be found if we impose the Siegel gauge condition

$$(b_0 + \tilde{b}_0)|\Psi\rangle = 0. \quad (3.3)$$

Since the exactness of the Virasoro generator in the BRST cohomology

$$\{Q_B + \tilde{Q}_B, b_0 + \tilde{b}_0\} = L_0 + \tilde{L}_0, \quad (3.4)$$

we get the equation of motion in the Siegel gauge

$$2(L_0 + \tilde{L}_0)|\Psi\rangle = (b_0 + \tilde{b}_0)|B_p\rangle. \quad (3.5)$$

Now using the expressions for the boundary state $|B_p\rangle$, we see that the source term $(b_0 + \tilde{b}_0)|B\rangle$ for the closed string field, due to $(b_0 + \tilde{b}_0)(c_0 + \tilde{c}_0)|\downarrow\rangle = 2|\downarrow\rangle$, is

$$\begin{aligned} & \int \frac{dk_0}{2\pi} \frac{dk_1}{2\pi} \frac{d^{D-p-2}k_\perp}{(2\pi)^{D-p-2}} U_{X^0}(k_0)|k_0\rangle \otimes U_{X^1}(k_1)|k_1\rangle \\ & \otimes \exp \left[- \sum_{n=1}^{\infty} \frac{1}{n} S_{ij} \alpha_{-n}^i \tilde{\alpha}_{-n}^j \right] |k_i\rangle \otimes \exp \left[\sum_{n=1}^{\infty} (c_{-n} \tilde{b}_{-n} - b_{-n} \tilde{c}_{-n}) \right] |\downarrow\rangle \\ \equiv & \int \frac{dk_0}{2\pi} \frac{dk_1}{2\pi} \frac{d^{D-p-2}k_\perp}{(2\pi)^{D-p-2}} U_{X^0}(k_0) U_{X^1}(k_1) \sum_{N=0}^{\infty} \hat{O}_N |k_0, k_1, k_\perp, k_\parallel = 0; \downarrow\rangle. \end{aligned} \quad (3.6)$$

Here we group the various descendant states according to their level, and introduce the operator \hat{O}_N to map the primary state to a level N state. This operator will in general depend on the momentum k_1 , due to the nontriviality of the Liouville theory along the X^1 direction.

To solve the closed string field $|\Psi\rangle$ produced by the brane decay, we make the following ansatz

$$|\Psi\rangle = \int \frac{dk_0}{2\pi} \frac{dk_1}{2\pi} \frac{d^{D-p-2}k_\perp}{(2\pi)^{D-p-2}} U_{X^1}(k_1) \sum_{N=0}^{\infty} \hat{O}_N \phi_N(k) |k, \downarrow\rangle, \quad (3.7)$$

where the symbol k is a abbreviation of $(k_0, k_1, k_\perp, k_\parallel = 0)$, and $\phi_N(k)$ is the unknown function to be determined. To calculate the action of L_0 and \tilde{L}_0 on the closed string field $|\Psi\rangle$, we can use the general argument in CFT to get the result, without knowing the detail form of the descendant operator \hat{O}_N introduced above. Consider a Ishibashi state $|h\rangle\rangle$ associated to the primary state $|h\rangle$, which can be written as

$$|h\rangle\rangle = \sum_{N=0}^{\infty} \hat{O}_N |h\rangle. \quad (3.8)$$

The index N denotes the level, and the state $\hat{O}_N |h\rangle$ is a combination of level N descendant states. It is not difficult to see that

$$L_0 \hat{O}_N |h\rangle = \left(\sum_i h^i + N \right) \hat{O}_N |h\rangle. \quad (3.9)$$

The action of \tilde{L}_0 is similar. Then we obtain the result of $2(L_0 + \tilde{L}_0)|\Psi\rangle$ as following

$$\sum_{N=0}^{\infty} \int \frac{dk_0}{2\pi} \frac{dk_1}{2\pi} \frac{d^{D-p-2}k_\perp}{(2\pi)^{D-p-2}} \delta^p(k_\parallel) U_{X^1}(k_1) [k^2 + V^2 + 4(N-1)] \phi_N(k) \hat{O}_N |k, \downarrow\rangle. \quad (3.10)$$

Insert the above formula and (3.6) into the Siegel gauge EOM (3.5) we have

$$[k^2 + V^2 + 4(N-1)] \phi_N(k) = U_{X^0}(k_0). \quad (3.11)$$

Here we have dropped the factor $\delta^p(k_\parallel)$ and restricted to $k_\parallel = 0$. So we see that although the X^1 direction is a nontrivial Liouville CFT, the result is simple: The equation we need

to determine the closed string field is just the same as the usual free CFT rolling tachyon, except the contribution, the V^2 term, from the linear dilaton. Having the closed string field in momentum space, we make the Fourier transformation with respect to k_0 to get the $\phi_N(x^0, \mathbf{k})$ as

$$\begin{aligned} \phi_N(x^0, \mathbf{k}) &= \int \frac{dk_0}{2\pi} \frac{U_{X^0}(k_0)}{-k_0^2 + \omega_{\mathbf{k}}^2} e^{ik_0 x^0} \\ &= \frac{iC}{\omega_{\mathbf{k}}} \tilde{\lambda}^{i\omega_{\mathbf{k}}/\beta} \Gamma(-i\omega_{\mathbf{k}}/\beta) \Gamma(1 + i\beta\omega_{\mathbf{k}}) e^{-i\omega_{\mathbf{k}} x^0}, \end{aligned} \quad (3.12)$$

where C is an unimportant constant, $\omega_{\mathbf{k}} = \mathbf{k}^2 + V^2 + 4(N-1)$, $\tilde{\lambda} = 2\pi\lambda/\Gamma(1 + \beta^2)$. This is the negative frequency solution. The positive one can be obtained by the replacement $\omega \rightarrow -\omega$. This result can be formally argued by use of the residue theorem. The positive and negative frequency parts correspond to the different choice of the contour. However the behavior of the integration along the large semicircle is not easy to analyze. In appendix we give a direct calculation of this Fourier transformation.

4. Closed string emission from the brane decay

In this section we calculate the closed string emission following the method of [9]. We first calculate the open string partition function, which is just the product of each directions. The optical theorem tells us that the imaginary part of this partition is the closed string emission rate.

4.1 Partition function

The total open string partition function can be written as

$$Z(t) = \langle B_p | \tilde{q}^{L_0 + \tilde{L}_0 - \frac{D-2}{12}} | B_p \rangle, \quad \tilde{q} = e^{-4t}. \quad (4.1)$$

In this boundary state formalism, the annulus is viewed as the propagation of a closed string, so the closed string Hamiltonian $L_0 + \tilde{L}_0 - \frac{c}{12}$ appears in the exponential. Note that we have included the ghost contribution which cancel two of the total directions. This partition function, of course, factorizes into the part of each direction. The nontrivial ones are the X^0 and X^1 directions, the remaining parts, including the ghost, are standard. The X^0 part of the partition function is [9]

$$Z_{X^0}(t) = \frac{\pi}{2} \int_{-\infty}^{\infty} dk_0 \frac{\chi_{(k_0 + iV_0)/2}(t)}{\sinh(\pi\beta k_0) \sinh(\pi k_0/\beta)}, \quad (4.2)$$

$$\chi_{\alpha}(t) = \eta(2it/\pi)^{-1} \tilde{q}^{-(\alpha - iV_0/2)^2}. \quad (4.3)$$

Here χ_{α} is the character. After a modular transformation to the open string channel and integrating over k_0 , the partition function becomes

$$Z_{X^0}(t) = \sqrt{2} \eta(i\pi/2t)^{-1} \int_{-\infty}^{\infty} d\nu q^{\nu^2} \frac{\partial f_{\beta}}{\partial \nu}(\nu), \quad q = e^{-\pi^2/t}. \quad (4.4)$$

where the function $f_\beta(\nu)$ is defined as

$$f_\beta(\nu) = \frac{1}{2} \int_{-\infty}^{\infty} \frac{dk_0}{k_0} \left[\frac{\sinh(\nu k_0)}{\sinh(\beta k_0) \sinh(k_0/\beta)} - \frac{\nu}{k_0} \right]. \quad (4.5)$$

Actually $f_\beta(\nu)$ here is just the special function $\log S_\beta(\frac{\beta+1/\beta}{2} - \nu)$, which is widely used in the literature of Liouville theory. The derivative of f has simple poles at

$$\nu = \left(m + \frac{1}{2}\right)\beta + \left(n + \frac{1}{2}\right)/\beta, \quad m, n \in \mathbb{Z}. \quad (4.6)$$

The integration contour is chosen to go below the real axis for negative ν and above for positive ν . For the calculation of the closed string emission rate it is needed to know the imaginary part of Z_{X^0} , which arises when going around the poles of $f_\beta(\nu)$

$$\text{Im}Z_{X^0}(t) = 2\sqrt{2}\pi \eta(i\pi/2t)^{-1} \sum_{m,n=0}^{\infty} e^{-[(n+\frac{1}{2})\beta+(m+\frac{1}{2})/\beta]^2\pi^2/t}. \quad (4.7)$$

Now we turn to the partition function of the X^1 direction. The ZZ brane [27] in this direction contains only the identity operator and its descendant fields. It corresponds to a degenerate representation of the Virasoro algebra. For general b there is only one null state at level one, so the character reads simply as

$$\chi_{ZZ}(q) = \eta(i\pi/2t)^{-1} \left[q^{-(b+1/b)^2/4} - q^{-(b-1/b)^2/4} \right], \quad q = e^{-\pi^2/t}. \quad (4.8)$$

The partition function of the X^1 direction, in the open string channel, is just the corresponding character

$$Z_{X^1}(t) = \eta(i\pi/2t)^{-1} \left[q^{-(b+1/b)^2/4} - q^{-(b-1/b)^2/4} \right], \quad (4.9)$$

since there is only one conformal family which corresponds to the identity operator. This Z_{X^1} is real, so it contributes to the imaginary part of the total partition function just a multiplicative factor. The remaining spacial part and the ghost part of the partition function is standard

$$Z_{X^i, \text{gh}}(t) = \frac{V_p}{2\sqrt{2}\pi t} \int \frac{d^p k_{\parallel}}{(2\pi)^p} q^{k_{\parallel}^2} \eta(i\pi/2t)^{4-D}. \quad (4.10)$$

Therefore the total partition function, the product of each part, reads as

$$\begin{aligned} Z(t) &= Z_{X^0}(t) Z_{X^1}(t) Z_{X^i, \text{gh}}(t) \\ &= \frac{V_p}{2\pi t} \eta(i\pi/2t)^{2-D} \left[q^{-(b+1/b)^2/4} - q^{-(b-1/b)^2/4} \right] \int_{-\infty}^{\infty} d\nu \int \frac{d^p k_{\parallel}}{(2\pi)^p} q^{\nu^2+k_{\parallel}^2} \frac{\partial f_\beta}{\partial \nu}(\nu). \end{aligned} \quad (4.11)$$

The imaginary part of this partition function is

$$\begin{aligned} \text{Im}Z(t) &= \frac{V_p}{t} \eta(i\pi/2t)^{2-D} \left[q^{-(b+1/b)^2/4} - q^{-(b-1/b)^2/4} \right] \\ &\quad \times \int \frac{d^p k_{\parallel}}{(2\pi)^p} q^{k_{\parallel}^2} \sum_{m,n=0}^{\infty} e^{-[(n+\frac{1}{2})\beta+(m+\frac{1}{2})/\beta]^2\pi^2/t}. \end{aligned} \quad (4.12)$$

4.2 Closed string emission

In this subsection we will calculate the closed string emission rate by use of the optical theorem. The annulus diagram can be cut open along a circle, the unitarity tells us that $\text{Im}\mathbf{Z}$, the imaginary part of the annulus amplitude, is just the closed string emission rate \bar{N} , which is what we want to know. The imaginary part of the annulus amplitude can be easily obtained from the CFT result in the previous subsection by integrating the moduli, since the (perturbative) string theory is just the world-sheet CFT coupled to the 2d gravity.

$$\begin{aligned} \text{Im}\mathbf{Z} = V_p \int_0^\infty \frac{ds}{s} \frac{1}{(4\pi s)^{p/2}} \left[e^{s(b+1/b)^2/4} - e^{s(b-1/b)^2/4} \right] \eta(is/2\pi)^{2-D} \\ \times \sum_{m,n=0}^\infty e^{-s[(n+\frac{1}{2})\beta+(m+\frac{1}{2})/\beta]^2}. \end{aligned} \quad (4.13)$$

Here we have integrated out the longitudinal momentum k_{\parallel} , and made a coordinate transformation $t = \pi^2/s$. The variable s is the world-sheet time of the open string, while t is that of the closed string.

Now we analyze the potential divergence of the emission rate $\text{Im}\mathbf{Z}$. For the limit $s \rightarrow \infty$, which is the open string IR and closed string UV, the integrand becomes

$$\frac{V_p}{s} \frac{1}{(4\pi s)^{p/2}} e^{\frac{1}{4}(b+1/b)^2 s} e^{-\frac{1}{4}(\beta+1/\beta)^2 s} e^{\frac{D-2}{24}s} \quad (4.14)$$

The world-sheet Weyl invariance tells us

$$26 = D + 6(-V_0^2 + V_1^2), \quad V_0 = \beta - \frac{1}{\beta}, \quad V_1 = b + \frac{1}{b}. \quad (4.15)$$

So we have

$$\frac{1}{4} \left(b + \frac{1}{b} \right)^2 - \frac{1}{4} \left(\beta + \frac{1}{\beta} \right)^2 + \frac{D-2}{24} = 0. \quad (4.16)$$

All of the exponential factors disappear. Therefore the integration of (4.14) at the neighborhood of infinity is convergent for all larger p except $p = 0$. Since this conclusion directly follows from the world-sheet Weyl invariance, it does not matter whether the dilaton is spacelike, timelike or lightlike.

There is another way to understand this facts from the picture of the Euclidean D-brane. Notice that the expression (4.13) can be viewed as the partition function of open strings stretched between an array of Euclidean D-branes along imaginary time [8, 33], i.e.

$$\bar{N} = \text{Im}\mathbf{Z} = V_p \left\langle B_- \left| \frac{b_0^+ c_0^+}{L_0 + \tilde{L}_0} \right| B_+ \right\rangle, \quad (4.17)$$

where the boundary state $|B_+\rangle$ describes branes located at imaginary time $X^0 = i(n + \frac{1}{2})\beta$ for $n \geq 0$, while $|B_-\rangle$ denotes branes at $X^0 = -i(m + \frac{1}{2})/\beta$ for $m \geq 0$. The other directions of these boundary state are the tensor product of the ZZ brane with the free part. A closed string UV divergence relates, through the modular transformation, with the open string

IR divergence. From this open string point of view the equation (4.16) just says that the lightest state of the open strings, stretched along imaginary time direction, is massless.

$$M_{\text{open}}^2 = \left(\frac{\beta}{2} + \frac{1}{2\beta}\right)^2 - \frac{1}{4} \left(b + \frac{1}{b}\right)^2 - \frac{D-2}{24} = 0. \quad (4.18)$$

The first term is the energy due to the finite length between two closest D-branes. The second term comes from the Liouville direction. The last term is just the zero point energy.

To calculate the emitted energy we use the formula, presented in [8],

$$\bar{E} = \frac{\partial}{\partial a} \left\langle B \left| \left[\frac{b_0^+ c_0^+}{L_0 + \tilde{L}_0} \right]_{\text{ret}} \right| B(a) \right\rangle_{a=0}. \quad (4.19)$$

It is not difficult to find that the emitted energy is finite for $p > 2$, while infinite for $p \leq 2$. This result is the same as that of [8], while not the same as [9]. In [9] although the linear dilaton is also tuned on, the moduli integration in the closed string UV region is convergent exponentially. This will raise a question. The original brane tension is proportional to the inverse of the string coupling, so the energy carried by the closed strings, which is finite, is insufficient by a power of g_s in the weak coupling limit. The model studied here exhibits a same Hagedorn behavior as [8], due to the presence of the Liouville direction, which contributes a exponentially increasing factor $e^{s(b+1/b)^2/4}$. We focus on the particular case $p = 0$, or the p -brane with all extended spacial direction compactified on circles. In this case the closed string emission rate diverges logarithmically, and the emitted energy diverges linearly. It is natural to chose the cutoff at $1/g_s$. Then we see that the emitted energy has the same order of magnitude as the original brane. The same conclusion as [8] follows: all of the brane energy converts into outgoing closed strings, and most of the energy is carried by closed strings of mass $\sim 1/g_s$. These divergence seems to invalidate the classical open string results. However we know that there is a new kind of open/closed string duality [10] in the process of open closed string tachyon condensation, according to which the complete dynamics of an unstable D-brane is captured by the quantum open string theory without any need to explicitly consider the coupling of the system to closed strings. In the context of the two-dimensional string theory, this duality can be checked more explicitly using the dual matrix model. In the case studied here it is, however, difficult to see it directly. However we believe that it is still right.

Next we go to the closed string IR region: $s \rightarrow 0$. Consider the following integral

$$\begin{aligned} & \sum_{m,n=0}^{\infty} \int_0^\delta \frac{ds}{s} \frac{1}{(4\pi s)^{p/2}} \left[e^{s(b+1/b)^2/4} - e^{s(b-1/b)^2/4} \right] e^{-sA_{mn}^2} \eta(is/2\pi)^{2-D} \\ & \sim \sum_{m,n=0}^{\infty} \int_0^\delta \frac{ds}{s} \frac{s}{(4\pi s)^{p/2}} e^{-sA_{mn}^2} \eta(is/2\pi)^{2-D}, \quad s \rightarrow 0, \end{aligned} \quad (4.20)$$

where we have defined $A_{mn} \equiv (n + \frac{1}{2})\beta + (m + \frac{1}{2})/\beta$. To analyze this expression it is convenient to take a modular transformation of the Dedekind η -function to obtain the following asymptotic behavior as $s \rightarrow 0$:

$$\eta(is/2\pi)^{2-D} \sim \left(\frac{s}{2\pi}\right)^{\frac{D-2}{2}} \left(e^{\frac{(D-2)\pi^2}{6s}} + (D-2) e^{\frac{(D-26)\pi^2}{6s}} + \dots \right). \quad (4.21)$$

The first term corresponds to the closed string tachyon which is an artifact of our bosonic string model and is absent in the superstring theory, so, as usual, we simply ignore it. Insert the second term into (4.20) we have

$$\sum_{m,n=0}^{\infty} \int_0^{\delta} \frac{ds}{(4\pi s)^{p/2}} \left(\frac{s}{2\pi}\right)^{\frac{D-2}{2}} (D-2) e^{\frac{(D-26)\pi^2}{6s}} e^{-sA_{mn}^2}. \quad (4.22)$$

When $D > 2$, the above quantity is smaller than the following one with $\delta \rightarrow \infty$

$$I = \sum_{m,n=0}^{\infty} \int_0^{\infty} \frac{ds}{(4\pi s)^{p/2}} \left(\frac{s}{2\pi}\right)^{\frac{D-2}{2}} (D-2) e^{\frac{(D-26)\pi^2}{6s}} e^{-sA_{mn}^2}, \quad (4.23)$$

since the integrand is positive. The infinite integral in I , after some trivial rescaling, can be related to the modified Bessel function $K_{\nu}(z)$, which has the following integral representation

$$K_{\nu}(z) = \frac{1}{2} \left(\frac{z}{2}\right)^{\nu} \int_0^{\infty} s^{-\nu-1} \exp\left(-s - \frac{z^2}{4s}\right) ds. \quad (4.24)$$

Then we have

$$I = (D-2) C \sum_{m,n=0}^{\infty} A_{mn}^{-(D-p)/2} K_{(D-p)/2}(z_{mn}), \quad (4.25)$$

$$z_{mn} = \sqrt{\frac{2(26-D)}{3}} \pi A_{mn}, \quad (4.26)$$

where C is an unimportant factor. To analyze the series I to be convergent or not, we need to know the behavior of the summand when m, n is large. Notice that when $m, n \rightarrow \infty$, z_{mn} also tends to infinity. Use the asymptotic expansion $K_{\nu}(z) \sim C z^{-1/2} \exp(-z)$ we know that the summand behaves as

$$A_{mn}^{-(D-p-1)/2} \exp\left[-\pi \sqrt{\frac{2(26-D)}{3}} A_{mn}\right]. \quad (4.27)$$

So the series I is finite. Therefore we have proved that, for $2 < D < 26$, there is no divergence in the emission rate when going to the closed string IR region. This result is similar with [9], since the spacial Liouville part of the partition function is not important in the closed string IR region, while the time direction, described by TBL, dominates here.

For the case of null dilaton, the dimension of the spacetime is 26. The IR behavior in the closed string channel is completely different from that of the spacelike dilaton studied above. Set $D = 26$ in (4.22), we need to estimate the following quantity

$$24 \sum_{m,n=0}^{\infty} \int_0^{\delta} \frac{ds}{(4\pi s)^{p/2}} \left(\frac{s}{2\pi}\right)^{12} e^{-sA_{mn}^2}. \quad (4.28)$$

The integral is essentially the incomplete Gamma function

$$\gamma(\alpha, x) = \int_0^x e^{-t} t^{\alpha-1} dt, \quad (4.29)$$

so (4.28) is equal to

$$\text{Const.} \times \sum_{m,n=0}^{\infty} A_{mn}^{-(26-p)/4} \gamma\left(\frac{26-p}{2}, A_{mn}^{1/2}\delta\right). \quad (4.30)$$

We first take the limit $\delta \rightarrow 0$. By use of the expansion $\gamma(\alpha, x) \sim \frac{1}{\alpha} x^\alpha$ as $x \rightarrow 0$, the quantity (4.28) tends to

$$\text{Const.} \times \sum_{m,n=0}^{\infty} 1, \quad (4.31)$$

which is badly divergent. This result is reasonable, since when $D = 26$ we have no more the exponential factor $e^{(D-26)\pi^2/6s}$ in (4.22), which suppress the integrand greatly as $s \rightarrow 0$. Physically when the dilaton is null, not only is the field configuration on the brane time-dependent, the background in the bulk is also variant along with time. To some extent the IR divergence we just find reflects this double time dependence.

5. Concluding remarks

In presence of the linear dilaton a more natural treatment is to embed it into the Liouville field theory, especially when studying the behavior of D-branes. It is impossible to impose the usual Dirichlet boundary condition in the direction where the linear dilaton turned on, since it is incompatible with the world-sheet conformal invariance. While when embedding into the Liouville theory, we can talk about the extended FZZT brane and the localized ZZ brane. The latter one is more interesting. The open string dynamics on it gives a holographic description of the bulk physics in two-dimensional string theory.

In this paper we consider the decay of the ZZ-type Dp -brane in the linear dilaton background with a Liouville potential switched on. This kind of branes satisfy the ZZ boundary condition in the Liouville direction and usual Dirichlet or Neumann in other spacial directions. We calculate the closed string field produced by the brane decay, and also analyze the emission rate of closed strings during this decay process. We find that when $2 < D < 26$ (spacelike dilaton) there is a Hagedorn behavior in the closed string UV region, as same as both in 26d and 2d string theory in this region. In the case of $p = 0$ (or the p -brane with all extended spacial directions wrapped on circles), the energy of the original brane completely converts into the outgoing closed strings. Due to the presence of the Liouville direction our result is different from that of [9], although both have a linear dilaton background. On the other hand, when going to the closed string IR, the emission rate is finite. In this region the time direction CFT dominates. For the case of null dilaton, the UV behavior does not change. In the IR region the result is, however, divergent.

There are some future directions. It is interesting to study the same question from the viewpoint of the effective dynamics. This may give us more insight into it. We can also turn on some electric or magnetic fluxes on the brane and see what happens. The electromagnetic field on the brane induces the conserved charges. The conservation law provides some constraints to the process, making it more controllable. It is also possible

to compactify some spacial directions and to study the effects of the winding closed strings emitted out from the unstable brane.

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A. Fourier transformation of the closed string field

In this appendix we give the direct calculation of the following Fourier transformation

$$I = \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \frac{\Gamma(ik_0/\beta) \Gamma(1 - i\beta k_0)}{-k_0^2 + \omega^2} \tilde{\lambda}^{-ik_0/\beta} e^{ik_0 x^0}. \quad (\text{A.1})$$

Using the Schwinger proper time

$$-\frac{1}{k_0^2 - \omega^2} = i \int_0^{\infty} dt e^{it(k_0^2 - \omega^2)}, \quad (\text{A.2})$$

and the integral representation of Gamma function, we have

$$\begin{aligned} I &= i \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \tilde{\lambda}^{-ik_0/\beta} e^{ik_0 x^0} \int_0^{\infty} dt e^{it(k_0^2 - \omega^2)} \int_0^{\infty} ds e^{-s} s^{-1 + ik_0/\beta} \int_0^{\infty} ds' e^{-s'} s'^{ik_0/\beta} \\ &= i \iint ds ds' e^{-s-s'} s^{-1} \int_0^{\infty} dt e^{-i\omega^2 t} \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} e^{itk_0^2 + ivk_0}, \end{aligned} \quad (\text{A.3})$$

where $v = x^0 - \frac{1}{\beta} \log \tilde{\lambda} + \frac{1}{\beta} \log s - \beta \log s'$. The integration over k_0 , by completing the square, is the Fresnel integration, and can be work out. Then

$$I = iC \iint ds ds' e^{-s-s'} s^{-1} \int_0^{\infty} t^{-1/2} \exp\left(-i\omega^2 t - \frac{iv^2}{4t}\right) dt. \quad (\text{A.4})$$

We do not care about the numerical factor, and just write it as C . Fortunately the integration over t , by some trivial rescaling of t , is just the integral representation of Hankel function with order minus one-half

$$\int_0^{\infty} t^{-1/2} \exp\left(-i\omega^2 t - \frac{iv^2}{4t}\right) dt = i\pi e^{-i\pi/4} \sqrt{\frac{v}{2\omega}} H_{-\frac{1}{2}}(-\omega v) = \frac{C}{\omega} e^{-i\omega v}. \quad (\text{A.5})$$

Now the integration I can be completely worked out as

$$I = \frac{iC}{\omega} \tilde{\lambda}^{i\omega/\beta} \Gamma(-i\omega/\beta) \Gamma(1 + i\beta\omega) e^{-i\omega x^0}. \quad (\text{A.6})$$

Of course we can use another Schwinger proper time representation, different from (A.2), as follows

$$-\frac{1}{k_0^2 - \omega^2} = -i \int_0^{\infty} dt e^{-it(k_0^2 - \omega^2)}, \quad (\text{A.7})$$

The corresponding result is similar, with the replacement $\omega \rightarrow -\omega$,

$$I' = \frac{iC}{\omega} \tilde{\lambda}^{-i\omega/\beta} \Gamma(i\omega/\beta) \Gamma(1 - i\beta\omega) e^{i\omega x^0}. \quad (\text{A.8})$$

These two result, I and I' , correspond to the different choices of the boundary conditions. One is the negative frequency solution, the other is the positive one.

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